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Collective modes in nonrelativistic electron-positron plasmas

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The longitudinal and transverse collective modes in a nonrelativistic electron-positron plasma are studied in two cases: a static uniform magnetic field is present or absent. The dispersion relations for the longitudinal and transverse collective modes in the absence of a magnetic field and those for the longitudinal mode in the presence of a magnetic field (Bernstein mode) are found to be similar to those for the one-component electron plasma. The transverse modes in the presence of a magnetic field, on the other hand, are found to be quite different from the electron-ion plasma: The dispersion relations for the left- and right-circularly polarized waves propagating parallel to the magnetic field are found to be identical. In addition to the transverse plasma oscillations, the low-frequency Alfvén mode exists, while the whistler mode does not exist. For waves propagating perpendicular to the magnetic field, the extraordinary wave becomes a pure transverse mode. In the cold-plasma limit, there is only one resonance at the cyclotron frequency and one cutoff frequency for the extraordinary mode, in contrast to the electron-ion plasma, where there are two hybrid resonances and two cutoff frequencies.

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I. INTRODUCTION

Recent experiments have opened up the possibility of creating a nonrelativistic electron-positron plasma in the laboratory. There are at least two schemes in which the nonrelativistic electron-positron plasma can be produced in the laboratory. In one scheme, a relativistic electron beam impinges on a high-Z target, where positrons are produced copiously. The relativistic pair plasma is then trapped in a magnetic mirror and is expected to cool rapidly by radiation [1]. In another scheme, positrons are accumulated from a radioactive source [2,3]. The purpose of the present paper is to study the collective modes in a nonrelativistic electron-positron (e^-e^+) plasma by deriving the dispersion relations and damping rates in two cases when a static spatially uniform magnetic field is absent or present. We consider only a homogeneous neutral electron-positron plasma in thermal equilibrium.

In general, the study of the collective modes in a plasma is of importance from the diagnostic point of view, since the observation of the propagation characteristics of the wave modes may be used in order to determine the physical parameters in the plasma [4]. So far, the properties of the electron-positron plasma have been studied mostly in the relativistic regime in the astrophysical context. This is because pair production, which is one of the most effective means for producing an electron-positron plasma, involves high-energy processes under most astrophysical conditions, such as solar flares, pulsars, black holes, and the jet phenomena associated with active galactic nuclei [5,6].

Some of the unique features of the neutral e^-e^+ plasma may be stated as follows.

(i) Same dynamical properties for electrons and positrons: Owing to the same masses and electric charge magnitudes for the electron and the positron, their dynamical behavior is the same. This is to be contrasted with the electron-ion (e-i) or electron-hole (e-h) plasma. The dynamical time scales are different from those in e-i and e-h plasmas. In the case of the e-i plasma, for example, the relation among the electron-electron (ee), ion-ion (ii), and electron-ion (ei) relaxation time scales is [7]

$$\tau_{ee}:\tau_{ii}:\tau_{ei} \cong 1:(m_i/m_e)^{1/2}:m_i/m_e \ , \tag{1.1}$$

where m_i and m_e are the masses of the ion and electron, respectively. Due to this hierarchy of the time scales the e-i plasma may exist as a two temperature plasma where the electrons and ions are both in thermal equilibrium but at different temperatures T_e and T_i, respectively. For an e^-e^+ plasma, on the other hand, the electron-electron ($--$), positron-positron ($++$), and electron-positron ($-+$) relaxation time scales are comparable [8]:

$$\tau_{--}:\tau_{++}:\tau_{-+} \cong 1:1:\tfrac{1}{2} \ . \tag{1.2}$$

Thus it is not possible to produce an e^-e^+ plasma with each component in thermal equilibrium and either with $T_- \gg T_+$ or with $T_- \ll T_+$. Here, T_σ is the temperature of the species σ ($-$ for electrons and $+$ for positrons). That is, when an electron plasma in thermal equilibrium at temperature T_- is mixed with a positron plasma in thermal equilibrium at temperature T_+ ($\neq T_-$), thermal equilibrium for each component will not be attained until the whole e^-e^+ plasma reaches a thermal equilibrium state.

(ii) Coupling to electromagnetic waves in the presence of a magnetic field: In the presence of a magnetic field, the electron and positron perform a gyromotion at the same frequency ($|\Omega_-| = \Omega_+$) in opposite directions. This is to be contrasted to the case with the e-i plasma, where $\Omega_i \ll |\Omega_e|$. For a charge-neutral e^-e^+ plasma, i.e., when $n_- = n_+$, the plasma couples to the left- and right-circularly polarized waves equally, which is in contrast to the e-i plasma.

(iii) Annihilation processes:

$$e^- + e^+ \rightarrow 2\gamma, 3\gamma, \ldots . \quad (1.3)$$

In addition to the ordinary plasma processes, pair annihilation can take place in an e^-e^+ plasma, which also applies to the non-neutral case. The pair-annihilation processes are of particular importance in astrophysics since the γ rays produced give a clear signature of the presence of positrons in the astrophysical object. The annihilation-line γ-ray source found near the Galactic Center, which has recently been identified [9] as the x-ray source 1E1740.7–2942, has allowed an estimate of the number of positrons and the time variability, as well as the environment in which the annihilations take place [5,6,10–13].

(iv) Annihilation time scales versus time scales for collective oscillations: Under realistic conditions, the electron-positron plasma is well defined in the sense that its lifetime against pair annihilation is much larger than the characteristic time scales for collective oscillations. In order to illustrate this point, let us consider a neutral ($n_- = n_+$) electron-positron plasma. As a characteristic time scale for collective oscillations, one may take the plasma frequency (cf. Sec. II)

$$\omega_p = (8\pi n_- e^2/m_e)^{1/2} = 7.98\times 10^4 n_-^{1/2} \text{ s}^{-1} , \quad (1.4)$$

where the cgs units will be used throughout this paper with n_- the electron number density in cm^{-3}. The thermally averaged rate coefficient (rate divided by target density) for direct annihilation ($e^- + e^+ \rightarrow 2\gamma$), which dominates at temperatures $T > 7\times 10^5$ K, is [12]

$$R_a/n_- \equiv \langle \sigma_a v_{\text{rel}} \rangle = \pi r_e^2 c J(a)$$
$$= 7.48\times 10^{-15} J(a) \text{ cm}^3 \text{ s}^{-1} . \quad (1.5)$$

Here, v_{rel} is the relative velocity, $r_e \equiv e^2/m_e c^2$, c is the velocity of light, and σ_a is the annihilation cross section in the Born approximation [14], $\pi r_e^2 c/v_{\text{rel}}$, times a Coulomb correction factor [12], which, after thermal averaging, yields a dimensionless function $J(a)$ of a parameter $a \equiv (2\pi^2 \mathcal{R}/k_B T)^{1/2}$, where $\mathcal{R} = 1$ Ry. The function $J(a)$ is weakly temperature dependent, varying, e.g., $1.1 < J(a) < 81.3$ for $3.1\times 10^8 > T > 3\times 10^2$ K. At temperatures $T < 7\times 10^5$ K, on the other hand, positronium formation via radiative recombination dominates at a temperature dependent rate [12]. For example, the rate coefficient varies

$$R_{\text{Ps}}/n_- = 1.1\times 10^{-11} - 2.4\times 10^{-14} \text{ cm}^3 \text{ s}^{-1} \quad (1.6)$$

for $T = 3\times 10^2$–3×10^6 K. Once the positronium is formed, its annihilation depends on the spin of the ground state [15]: The parapositronium decays into two photons with a lifetime $\tau_0 = 1.24\times 10^{-10}$ s, while the ortho-positronium decays into three photons with a lifetime $\tau_1 = 1.39\times 10^{-7}$ s. From (1.4) and (1.5), one obtains

$$R_a/\omega_p = 0.937\times 10^{-19} J(a) n_- \quad (1.7)$$

at temperature T and

$$R_{\text{Ps}}/\omega_p = 6.36\times 10^{-17} n_-^{1/2} \quad (1.8)$$

at $T = 10^3$ K, for example. In either case, the electron-positron plasma will live sufficiently long for many collective oscillations before it annihilates.

The organization of the present paper is as follows. In Sec. II, we derive the dispersion relations for the longitudinal modes in an electron-positron plasma at finite temperatures in the cases where a magnetic field is absent or present. In the presence of a magnetic field, the longitudinal plasma wave and the Bernstein mode are considered. In Sec. III, the transverse modes are studied with or without a magnetic field. For the waves propagating parallel to the magnetic field, the dispersion relations for upper and lower branches are derived. For wave propagation perpendicular to the magnetic field, the dispersion relations for extraordinary and ordinary waves are derived in the cold-plasma limit. In Sec. IV, a summary and some discussion are given.

II. LONGITUDINAL MODES

The dielectric tensor may be obtained from the combination of the Maxwell equations and the linearized Vlasov equation [16–19]. We first consider the longitudinal modes in the plasma with and without an external magnetic field.

A. Without external magnetic fields: The Langmuir mode

The frequency- and wave-number-dependent longitudinal dielectric function is [20]

$$\epsilon(\mathbf{k},\omega) = 1 + \sum_{\sigma=-,+} (k_\sigma^2/k^2) W(\omega/k(k_B T_\sigma/m_\sigma)^{1/2}) , \quad (2.1)$$

where $W(x)$ is the plasma dispersion function [21], $k_\sigma \equiv (4\pi n_\sigma q_\sigma^2/k_B T_\sigma)^{1/2}$ the Debye wave number with n_σ the number density, q_σ the electric charge, T_σ the temperature, m_σ the mass for the particle species σ ($-$ for electrons and $+$ for positrons), and k_B the Boltzmann constant. In addition, $k \equiv |\mathbf{k}|$ and $m_- = m_+ \equiv m$. When the temperature of the electrons is the same as that of the positrons ($T_- = T_+ \equiv T$), (2.1) reduces to

$$\epsilon(\mathbf{k},\omega) = 1 + (k_D^2/k^2) W(\omega/k(k_B T/m)^{1/2}) , \quad (2.1a)$$

where $k_D^2 \equiv k_-^2 + k_+^2$. In order to find the dispersion relation for collective modes, we seek a solution to the equation $\epsilon(k,\omega) = 0$. A well-defined collective mode exists where the phase velocity of the wave is much larger than the thermal velocity of the particles, $\omega/k \gg (k_B T/m)^{1/2}$. In such a region, one finds

$$\omega(k) = \omega_p[1 + \tfrac{3}{2}(k/k_D)^2 + \cdots] , \quad (2.2a)$$

$$\gamma(k) = -(\pi/8)^{1/2} \omega_p (k_D/k)^3 \exp[-(k_D/k)^2/2 - \tfrac{3}{2}] , \quad (2.2b)$$

where $\omega_p^2 \equiv \omega_{p-}^2 + \omega_{p+}^2$ with $\omega_{p\sigma}^2 \equiv 4\pi n_\sigma q_\sigma^2/m_\sigma$. The dispersion relation (2.2) is quite similar to that for a one-component electron plasma. The latter may be trivially obtained from the former with the replacements $k_D^2 \to k_-^2, \omega_p^2 \to \omega_{p-}^2$.

B. In the presence of an external uniform magnetic field

In the presence of an external static uniform magnetic field of strength B, the longitudinal dielectric function is given by [20]

$$\epsilon(\mathbf{k},\omega)=1+\sum_{\sigma=-,+}(k_\sigma^2/k^2)\left\{1+\sum_{n=-\infty}^{\infty}[\omega/(\omega-n\Omega_\sigma)][W((\omega-n\Omega_\sigma)/|k_\parallel|(k_BT_\sigma/m_\sigma)^{1/2})-1]\Lambda_n(\beta_\sigma)\right\}, \quad (2.3)$$

where $\Omega_\sigma \equiv q_\sigma B/m_\sigma c$ is the cyclotron frequency including the sign of the electric charge q_σ (i.e., $\Omega_+=|e|B/m_+c\equiv\Omega>0$, $\Omega_-=-\Omega_+<0$), $\beta_\sigma\equiv k_\perp^2 k_BT_\sigma/m_\sigma\Omega_\sigma^2=(\omega_{p\sigma}^2/\Omega_\sigma^2)(k_\perp^2/k_\sigma^2)$, $\Lambda_n(\beta_\sigma)\equiv I_n(\beta_\sigma)\exp(-\beta_\sigma)$, and $I_n(x)$ is the modified Bessell function of the first kind with $k\equiv|\mathbf{k}|=(k_\parallel^2+k_\perp^2)^{1/2}$ and $k_\parallel\equiv\mathbf{k}\cdot\mathbf{B}/|\mathbf{k}|\,|\mathbf{B}|$. In the following, we consider the case $T_-=T_+\equiv T$ (i.e., $\beta_-=\beta_+\equiv\beta$). Then, noting that [22] $I_{-n}(\beta)=I_n(\beta)$, one finds that

$$\epsilon(\mathbf{k},\omega)=1+(k_D^2/k^2)\left\{1+\sum_{n=-\infty}^{\infty}[\omega/(\omega-n\Omega_-)][W((\omega-n\Omega_-)/|k_\parallel|(k_BT/m)^{1/2})-1]\Lambda_n(\beta)\right\}. \quad (2.3a)$$

1. *Longitudinal plasma wave*

When the magnetic field is weak such that $\Omega^2 \ll \omega_p^2$, the dispersion relation for Langmuir wave is little modified from its field free case. Let us study the longitudinal collective mode in the presence of a strong magnetic field ($\Omega^2 \gg \omega_p^2$). We look for a solution near $\omega=\pm\omega_p$ in the long-wavelength limit $k \ll k_D$ of the form

$$\omega=\pm\omega_0(\mathbf{k})+i\gamma_0(\mathbf{k}). \quad (2.4)$$

One finds

$$\omega_0(\mathbf{k})=(|k_\parallel|/k)\omega_p[\Lambda_0(\beta)]^{1/2}\cong(|k_\parallel|/k)\omega_p, \quad (2.5)$$

$$\gamma_0(\mathbf{k})/\omega_0(\mathbf{k})=-(\pi/8)^{1/2}(k_D/k)^3[\Lambda_0(\beta)]^{3/2} \times\exp[-(k_D^2/2k^2)\Lambda_0(\beta)]. \quad (2.6)$$

When $k_\perp=0$ (wave propagation parallel to the magnetic field), the dispersion relation and the damping rate of this mode [(2.5) and (2.6)] reduces to those of the Langmuir mode [(2.2)] in the magnetic field free case. The condition that the magnetic field be strong ($\Omega^2 \gg \omega_p^2$) may be physically stated in the following equivalent forms.

(1) The particles perform many Larmor motions during one plasma oscillation.

(2) The energy density of the magnetic field is much larger than the rest mass energy density of the particles. This is because $\Omega_-^2/\omega_{p-}^2=(B^2/4\pi)/n_-mc^2 \gg 1$.

(3) The Larmor radius (r_L) is much smaller than the Debye length (λ_D). This is because $|\Omega_-|/\omega_{p-}=\lambda_D/r_L \gg 1$.

2. *The Bernstein mode*

Next, we study the collective mode near the cyclotron harmonics [20,23]

$$\omega\cong n\Omega_- \quad (n=\pm1,\pm2,\ldots). \quad (2.7)$$

We look for a solution of the form

$$\omega=\omega_n(\mathbf{k})+i\gamma_n(\mathbf{k}) \quad (2.8a)$$

$$\equiv n\Omega_-[1+\Delta_n(\mathbf{k})]+i\gamma_n(\mathbf{k}), \quad (2.8b)$$

under the conditions that

$$n\Omega_-[1+\Delta_n(\mathbf{k})]<(n+1)\Omega_- \quad (2.9)$$

and

$$|\omega-n\Omega_-|/|k_\parallel|(k_BT/m)^{1/2} \gg 1. \quad (2.10)$$

The condition on $\Delta_n(\mathbf{k})$ from (2.9) and (2.10) is

$$|k_\parallel|(k_BT/m)^{1/2}/|n\Omega_-| \ll |\Delta_n(\mathbf{k})| < 1/n. \quad (2.11)$$

The condition (2.11) allows one to use the asymptotic form of the plasma dispersion function [21], which yields

$$\Delta_n(\mathbf{k})=k_D^2\Lambda_n(\beta)/\{k^2+k_D^2[1-\Lambda_0(\beta)]\}, \quad (2.12)$$

$$|\gamma_n(\mathbf{k})/\omega_n(\mathbf{k})| =(\pi/2)^{1/2}[|n\Omega_-|\Delta_n(\mathbf{k})^2/|k_\parallel|(k_BT/m)^{1/2}] \times\exp\{-[n\Omega_-\Delta_n(\mathbf{k})]^2/2|k_\parallel|^2(k_BT/m)\}. \quad (2.13)$$

One finds that there is no damping as $k_\parallel \to 0$ (i.e., $k_\perp \to k$). This is because the electron's motion is not free in the direction perpendicular to the magnetic field, so that the resonance condition cannot be satisfied when $k_\parallel=0$.

III. TRANSVERSE MODES

A. Without external magnetic fields

In the absence of an external magnetic field, the frequency- and wave-number-dependent transverse dielectric function for an isotropic electron-positron plasma at temperature $T\,(\equiv T_-=T_+)$ is [20]

$$\epsilon_T(k,\omega)=1-\sum_\sigma(\omega_{p\sigma}^2/\omega^2)[1-W(\omega/k(k_BT_\sigma/m_\sigma)^{1/2})] \quad (3.1a)$$

$$=1-(\omega_p^2/\omega^2)[1-W(\omega/k(k_BT/m)^{1/2})], \quad (3.1b)$$

where $\omega_p^2 \equiv \omega_{p-}^2 + \omega_{p+}^2$. Solving the equation $\epsilon_T(k,\omega) = (ck/\omega)^2$, one finds the dispersion relation for a transverse plasma mode of the form $\omega = \omega(k) + i\gamma(k)$ as

$$\omega(k)^2 = \omega_p^2 + c^2k^2 , \quad (3.2)$$

$$\gamma(k) = 0 . \quad (3.3)$$

The damping is absent because the phase velocity of the wave obtained from (3.2) is always greater than the velocity of light, so that no particles can be resonant with the wave. This result is analogous to the one-component electron plasma.

B. In the presence of an external magnetic field

We now consider the electron-positron plasma in an external static uniform magnetic field. The dispersion relation for the transverse collective modes may be obtained by solving the equation [20]

$$\det|\overleftrightarrow{\epsilon}(\mathbf{k},\omega) - (ck/\omega)^2(\overleftrightarrow{\mathrm{I}} - \mathbf{k}\otimes\mathbf{k}/k^2)| = 0 . \quad (3.4)$$

1. *Wave propagation parallel to the magnetic field*

We first study the wave propagating parallel to the magnetic field with the wave vector $\mathbf{k} = (0,0,k)$, where the z axis is chosen in the direction of the magnetic field. It is convenient to introduce a unitary matrix [20]

$$\overleftrightarrow{\mathrm{U}} = \begin{vmatrix} 1/\sqrt{2} & -i/\sqrt{2} & 0 \\ -i/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{vmatrix} , \quad (3.5)$$

where $\overleftrightarrow{\mathrm{U}}\overleftrightarrow{\mathrm{U}}^\dagger = \overleftrightarrow{\mathrm{U}}^\dagger\overleftrightarrow{\mathrm{U}} = \overleftrightarrow{\mathrm{I}}$. Then, the electric field component in the Cartesian coordinates may be transformed to

$$\begin{vmatrix} E_r \\ E_l \\ E_z \end{vmatrix} = \overleftrightarrow{\mathrm{U}} \begin{vmatrix} E_x \\ E_y \\ E_z \end{vmatrix} , \quad (3.6)$$

where

$$E_r = (1/\sqrt{2})(E_x - iE_y) , \quad (3.7)$$

$$E_l = (-i/\sqrt{2})(E_x + iE_y) . \quad (3.8)$$

Under this transformation, the dielectric tensor becomes [20]

$$\overleftrightarrow{\mathrm{U}}\overleftrightarrow{\epsilon}\overleftrightarrow{\mathrm{U}}^\dagger = \begin{vmatrix} \epsilon_r & 0 & 0 \\ 0 & \epsilon_l & 0 \\ 0 & 0 & \epsilon \end{vmatrix} , \quad (3.9)$$

where

$$\epsilon_r(k,\omega) = 1 - \sum_\sigma [\omega_{p\sigma}^2/\omega(\omega+\Omega_\sigma)][1 - W((\omega+\Omega_\sigma)/k(k_BT_\sigma/m_\sigma)^{1/2})] , \quad (3.10)$$

$$\epsilon_l(k,\omega) = 1 - \sum_\sigma [\omega_{p\sigma}^2/\omega(\omega-\Omega_\sigma)][1 - W((\omega-\Omega_\sigma)/k(k_BT_\sigma/m_\sigma)^{1/2})] , \quad (3.11)$$

$$\epsilon(k,\omega) = 1 + \sum_\sigma (k_\sigma^2/k^2)W(\omega/k(k_BT_\sigma/m_\sigma)^{1/2}) . \quad (3.12)$$

For the waves propagating parallel to the magnetic field, the dielectric functions simplify considerably with only the fundamental cyclotron frequency contributing. From the charge neutrality condition ($n_- = n_+$),

$$\begin{aligned}\epsilon_r(k,\omega) &= \epsilon_l(k,\omega) \\ &= 1 - [\omega_{p-}^2/\omega(\omega-\Omega)][1 - W((\omega-\Omega)/k(k_BT/m)^{1/2})] \\ &\quad - [\omega_{p-}^2/\omega(\omega+\Omega)][1 - W((\omega+\Omega)/k(k_BT/m)^{1/2})] ,\end{aligned} \quad (3.13)$$

where $\omega_{p-}^2 \equiv 4\pi n_- e^2/m = \omega_{p+}^2 \equiv 4\pi n_+ e^2/m$ and $\Omega \equiv |e|B/mc > 0$. In the region $|\omega - \Omega| \gg k(k_BT/m)^{1/2}$ [thus $\omega + \Omega \gg k(k_BT/m)^{1/2}$], (3.13) becomes

$$\epsilon_r(k,\omega) = 1 - \omega_p^2/(\omega^2 - \Omega^2) , \quad (3.14)$$

so that one obtains the dispersion relation

$$\omega^2 = \tfrac{1}{2}([\Omega^2 + \omega_p^2 + (ck)^2] \pm \{[\Omega^2 + \omega_p^2 + (ck)^2]^2 - 4(ck\Omega)^2\}^{1/2}) . \quad (3.15)$$

(a) *Upper branch.* Equation (3.15) expresses two solutions. Let us first consider the upper branch, with the plus sign in (3.15). In the long-wavelength limit ($ck \ll \Omega, \omega_p$), the dispersion relation becomes

$$\omega(k) = (\Omega^2 + \omega_p^2)^{1/2}[1 + \omega_p^2(ck)^2/2(\Omega^2 + \omega_p^2)^2 + \cdots] , \quad k \to 0 , \quad (3.16)$$

while in the short-wavelength limit ($ck \gg \Omega, \omega_p$)

$$\omega(k) = ck[1 + \tfrac{1}{2}(\omega_p/ck)^2 + \cdots], \quad k \to \infty . \quad (3.17)$$

The dispersion relation (ω vs k) is schematically shown in Fig. 1. When $\omega_p > \Omega$, (3.15) with the plus sign reduces to the magnetic field free case. In addition, Fig. 2 illustrates the dielectric function $\epsilon_l(k,\omega)$ [$= \epsilon_r(k,\omega) = (ck/\omega)^2$] versus frequency ω. From Fig. 2, one finds that there is a resonance at $\omega = \Omega$, and the wave is totally reflected when $\Omega < \omega < \omega_c$, where $\omega_c \equiv (\Omega^2 + \omega_p^2)^{1/2}$ is the cutoff frequency.

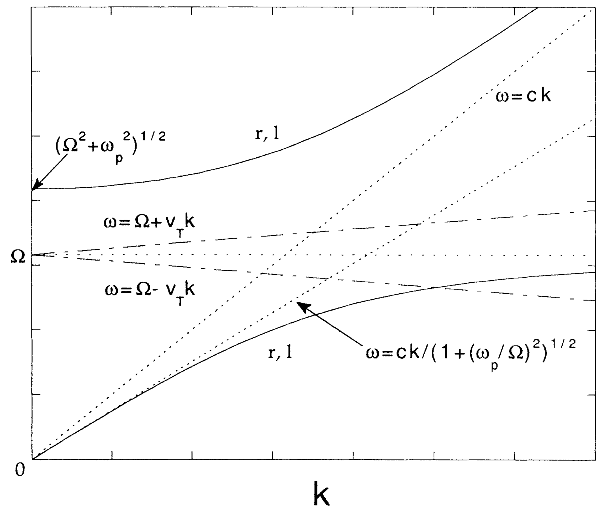

FIG. 1. The dispersion relation for the transverse collective mode propagating parallel to the magnetic field. The left (l) and right (r) circularly polarized waves have the same dispersion (solid curves). The lower branch is subject to damping due to Doppler-shifted cyclotron resonance at finite wave numbers (the portion of the lower solid curve between the dash-dotted lines).

(b) *Lower branch.* Let us now consider the lower branch, with the minus sign in (3.15). In the long-wavelength limit ($ck \ll \Omega, \omega_p$), the dispersion relation becomes

$$\omega(k) = ck/[1+(\omega_p/\Omega)^2]^{1/2}, \quad k \to 0 , \tag{3.18}$$

which is the Alfvén mode. In the short-wavelength limit ($ck \gg \Omega, \omega_p$), on the other hand, the dispersion relation becomes

$$\omega(k) = \Omega, \quad k \to \infty . \tag{3.19}$$

This collective mode is strongly damped when $|\omega - \Omega| < k(k_B T/m)^{1/2}$, i.e., when $\Omega - kv_T < \omega < \Omega + kv_T$ with $v_T \equiv (k_B T/m)^{1/2}$. This damping is due to the Doppler-shifted cyclotron resonance, where the frequency of the wave seen by the electron with the velocity component parallel to the magnetic field $v_\parallel(<v_T)$ is $\omega' = \omega \pm kv_\parallel$ with the plus (minus) sign corresponding to the wave propagating antiparallel (parallel) to $v_\parallel$. For the electron-positron plasma, the right-circularly polarized wave is resonant with the electrons and the left-circularly polarized wave is resonant with the positrons. The lower branch is also illustrated in Fig. 1. It is instructive to compare the transverse collective modes propagating parallel to the magnetic field in the electron-positron plasma with those in the one-component electron plasma and electron-ion plasma. One finds the following unique features of the electron-positron plasma.

(1) The dielectric function of the right-circularly polarized wave $\epsilon_r(k,\omega)$ is identical to that of the left-circularly polarized wave $\epsilon_l(k,\omega)$. This results from the fact that the electric charge to the mass ratio is equal in magnitude and opposite in sign for the electron and the positron.

(2) The resonance occurs for both the right- and left-circularly polarized waves. The electrons (positrons) are responsible for the resonance with the right- (left-) circularly polarized wave. This feature is also the direct consequence of the equal electric charge to mass ratio with opposite sign.

(3) The electron-positron plasma being a two-component plasma, the helicon mode does not exist, which is in contrast to the one-component electron plasma, while the Alfvén mode exists as in the case of the electron-ion plasma.

(4) Owing to the symmetry between the positively and negatively charged particles, the dispersion relation for the right-circularly polarized wave is identical to the left-circularly polarized wave. Therefore the whistler mode does not exist, which is in contrast to the electron-ion plasma.

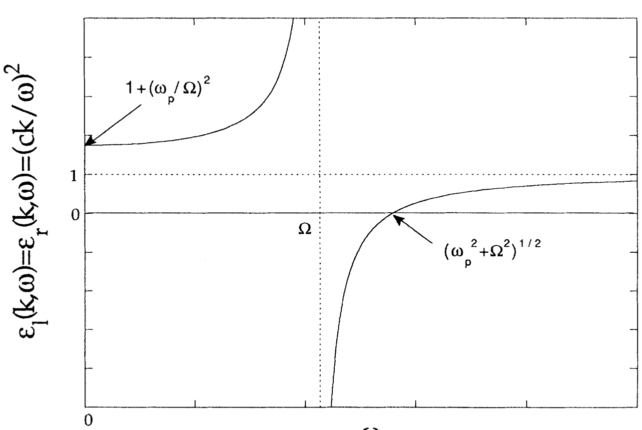

FIG. 2. The dielectric function $\epsilon_l(k,\omega)$ $[=\epsilon_r(k,\omega) =(ck/\omega)^2]$ as a function of the frequency ω. A cyclotron resonance occurs at $\omega = \Omega$, and the wave is totally reflected in the region $\Omega < \omega < \omega_c$, where $\omega_c \equiv (\Omega^2 + \omega_p^2)^{1/2}$ is the cutoff frequency.

2. *Wave propagation perpendicular to the magnetic field*

We consider the transverse collective modes whose wave vectors are perpendicular to the magnetic field. Without loss of generality, one may choose the x axis to be the direction of the wave vector, so that $\mathbf{k} = (k,0,0)$. In this case, the dielectric tensor takes the form [20]

$$\overleftrightarrow{\epsilon}(k,\omega) = \begin{vmatrix} \epsilon_1 & -i\epsilon_x & 0 \\ i\epsilon_x & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{vmatrix} , \tag{3.20}$$

where

$$\epsilon_1(k,\omega) = 1 - \sum_\sigma (k_\sigma^2/k^2) \sum_n [(n\Omega_\sigma)^2/\omega(\omega - n\Omega_\sigma)] \times \Lambda_n(\beta_\sigma) , \tag{3.21}$$

$$\epsilon_2(k,\omega)=1-\sum_\sigma (k_\sigma^2/k^2) \times \sum_n [(n\Omega_\sigma)^2/\omega(\omega-n\Omega_\sigma)] \times [\Lambda_n(\beta_\sigma)-(2\beta_\sigma^2/n^2)\Lambda_n'(\beta_\sigma)] , \tag{3.22}$$

$$\epsilon_3(k,\omega)=1-\sum_\sigma (\omega_{p\sigma}^2/\omega^2)\sum_n [(\omega/(\omega-n\Omega_\sigma)]\Lambda_n(\beta_\sigma) , \tag{3.23}$$

$$\epsilon_x(k,\omega)=\sum_\sigma (\omega_{p\sigma}^2/\omega^2)\sum_n [(\omega/(\omega-n\Omega_\sigma)]n\Lambda_n'(\beta_\sigma) . \tag{3.24}$$

One immediately notices that $\epsilon_x(k,\omega)=0$, since the electric charge to mass ratio is of the same magnitude with opposite sign for the electron and the positron. Therefore the dielectric tensor becomes diagonal. This feature is unique to the electron-positron plasma. For simplicity, let us consider the cold-plasma limit ($T\to 0$), where only the fundamental cyclotron frequency contributes. Then, the nonzero components of the dielectric tensor are

$$\epsilon_1(k,\omega)=\epsilon_2(k,\omega)=1-\omega_p^2/(\omega^2-\Omega^2) , \tag{3.25}$$

$$\epsilon_3(k,\omega)=1-\omega_p^2/\omega^2 . \tag{3.26}$$

(a) *Extraordinary wave.* The dispersion relation for the extraordinary wave may be obtained by solving the equation

$$(\epsilon_1\epsilon_2-\epsilon_x^2)/\epsilon_1=(ck/\omega)^2 . \tag{3.27}$$

Because $\epsilon_x=0$, (3.27) simplifies to

$$\epsilon_2(k,\omega)=(ck/\omega)^2 . \tag{3.28}$$

The electric-field components are $E_z=0$ and $E_x/E_y=i(\epsilon_x/\epsilon_1)=0$. Therefore $\mathbf{E}=(0,E_y,0)$, i.e., the extraordinary wave is a pure transverse wave such that $\mathbf{E}\perp\mathbf{k}$ and $\mathbf{E}\perp\mathbf{B}$. Contrary to the one-component electron plasma, the extraordinary wave in the electron-positron plasma is not a hybrid mode, since the longitudinal component of the electric field is absent ($E_x=0$). This is a unique feature of the electron-positron plasma. Solving (3.28) with (3.25), one finds that the dispersion relation for the extraordinary wave propagating perpendicular to the magnetic field is identical to that for the transverse wave propagating parallel to the magnetic field (3.15). The dispersion relation for the extraordinary wave propagating perpendicular to the magnetic field is illustrated for the two cases $\Omega>\omega_p$ and $\Omega>\omega_p$ in Fig. 3. It is instructive to compare the extraordinary waves in the electron-positron plasma and the electron-ion plasma in the cold-plasma limit. The comparison is made in Table I. The electron-ion plasma has two cutoff frequencies, while the electron-positron plasma has only one. In addition, the electron-ion plasma has two hybrid resonances, while the electron-positron plasma has only one (cyclotron) resonance. One can obtain the results for the electron-positron plasma by replacing the ion mass (m_i) by the positron mass (m_+). However, some care must be taken. By taking the limit $m_i\to m_+$, the upper (UH) and lower hybrid (LH) resonance frequencies in the electron-ion plasma reduce to $\omega_{\rm UH}\to(\omega_p^2+\Omega^2)^{1/2}$ and $\omega_{\rm LH}\to\Omega$, respectively. On the other hand, there is only one resonance (cyclotron resonance) $\omega=\Omega$ in the electron-positron plasma from (3.28). Obviously, this cyclotron resonance in the electron-positron plasma corresponds to the lower hybrid resonance in the electron-ion plasma. Apparently, no resonance exists in the electron-positron plasma that corresponds to the upper hybrid resonance in the electron-ion plasma. Therefore it appears that the limiting procedure $m_i\to m_+$ in the electron-ion plasma does not give correct results for the resonance in the electron-positron plasma. Let us now take a look at the cutoff frequencies. In the limit $m_i\to m_+$, the two cutoff frequencies in the electron-ion plasma merge into one

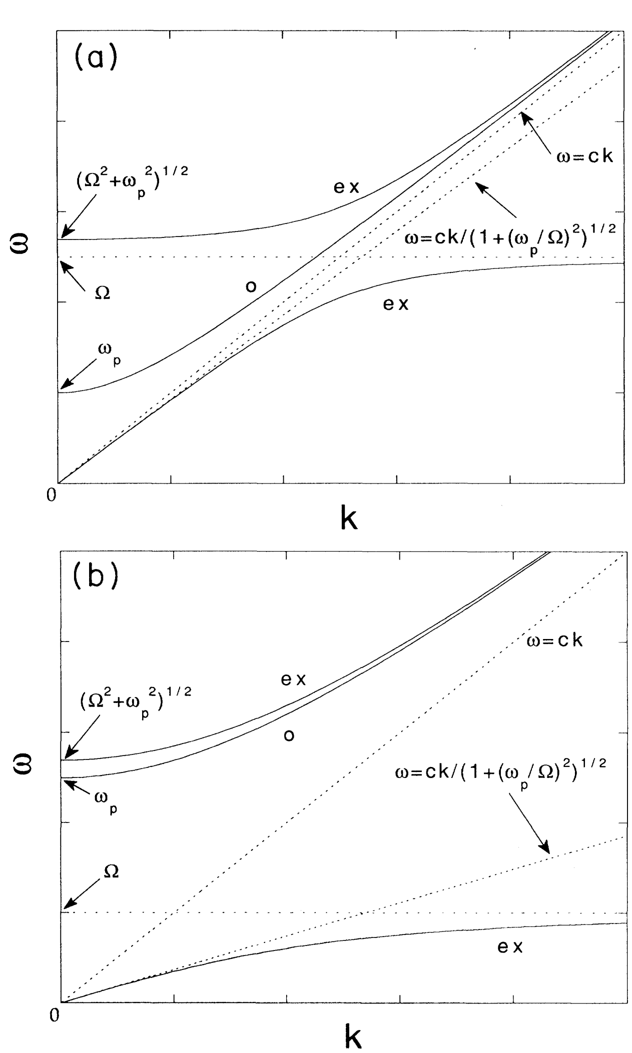

FIG. 3. The dispersion relation for the transverse collective modes propagating perpendicular to the magnetic field in the cold-plasma limit in the two cases where the magnetic field is (a) strong ($\Omega>\omega_p$) and (b) weak ($\Omega<\omega_p$). There are two extraordinary (ex) wave branches (upper and lower) and one ordinary (o) wave branch.

TABLE I. Comparison of the extraordinary waves in the electron-positron plasma and in the electron-ion plasma in the cold-plasma limit. Here, $\Omega_p^2 \equiv \omega_{pe}^2 + \omega_{pi}^2$, $\omega_e^2 \equiv \omega_{pe}^2 + \Omega_e^2$, $\omega_i^2 \equiv \omega_{pi}^2 + \Omega_i^2$, $\Omega \equiv \Omega_+ = -\Omega_-$, and $\omega_p^2 \equiv \omega_{p-}^2 + \omega_{p+}^2$.

Electron-ion plasma	Electron-positron plasma
Cutoff frequencies	
$\omega_c = [(\lvert\Omega_e\rvert + \Omega_i)^2/4 + \Omega_p^2]^{1/2} \pm (\lvert\Omega_e\rvert - \Omega_i)/2$	$\omega_c = (\Omega^2 + \omega_p^2)^{1/2}$
Resonance frequencies	
Upper and lower hybrid resonances (UH and LH)	Cyclotron resonance
$\omega_{\substack{\mathrm{UH}\\ \mathrm{LH}}}^2 = (\omega_e^2 + \omega_i^2)/2 \pm [(\omega_e^2 - \omega_i^2)^2/4 + \omega_{pe}^2 \omega_{pi}^2]^{1/2}$	$\omega = \Omega$

cutoff frequency $\omega_c = (\omega_p^2 + \Omega^2)^{1/2}$. Thus this limiting procedure does correctly reproduce the cutoff frequency in the electron-positron plasma. The origin of the discrepancy in the resonance frequencies may be explained as follows. For the electron-ion plasma there are two solutions each for the resonance and cutoff, which are respectively given by the zeros of the denominator and numerator of the dielectric function [19]. When the limit $m_i \rightarrow m_+$ is taken, there is a cancellation of a factor between the denominator and numerator of the dielectric function. Therefore there is now only one solution each for the resonance and cutoff. This is why another solution disappears in this limit.

(b) *Ordinary wave.* The dispersion relation for the ordinary wave is given by solving the equation $\epsilon_3(k,\omega) = (ck/\omega)^2$, which in the cold-plasma limit [Eq. (3.26)] yields $\omega^2 = \omega_p^2 + c^2k^2$. The only nonzero electric field component is E_z, so that $\mathbf{E} \parallel \mathbf{B}$. Therefore the dispersion relation is the same as the field-free case with the magnetic field having no effect. The dispersion relation is schematically shown in Fig. 3. For warm plasma, the higher cyclotron harmonics contribute to ϵ_1 and ϵ_2 (affecting the extraordinary wave) and to ϵ_3 (affecting the ordinary wave). For the ordinary wave, nonlocal effects become important at the intersections of $\omega = ck$ and $\omega = n\Omega$ with n a positive integer.

IV. SUMMARY AND DISCUSSION

The nonrelativistic electron-positron plasma sustains well-defined collective modes. The longitudinal collective modes both in the absence and presence of an external static uniform magnetic field and the transverse collective mode in the absence of a magnetic field are found to be analogous to those either in the one-component electron plasma or the electron-ion plasma. On the other hand, the transverse collective modes in the presence of a magnetic field are found to be quite different from those in the one-component electron plasma and electron-ion plasma.

When the charge neutrality condition is satisfied $(n_- = n_+)$, many of the unique features of the electron-positron plasma arise from the facts that the electric charge to mass ratio is the same in magnitude but opposite in sign for the electron and the positron. Due to this symmetry, it is found that the dielectric function of the left-circularly polarized wave $\epsilon_l(k,\omega)$ is identical to that of the right-circularly polarized wave $\epsilon_r(k,\omega)$ for the collective modes propagating parallel to the magnetic field. Therefore the dispersion relation for the left-circularly polarized wave is the same as that for the right-circularly polarized wave. Another consequence is that the whistler mode does not exist, which is in contrast to the electron-ion plasma.

The resonance occurs for both the right-circularly polarized waves (that resonate with the electrons) and the left-circularly polarized waves (that resonate with the positrons). Since the electron-positron plasma is a two-component plasma, the helicon mode does not exist, which is in contrast to the one-component electron plasma. On the other hand, the Alfvén mode exists as in the case of the electron-ion plasma.

For the collective modes propagating perpendicular to the magnetic field, the dielectric tensor becomes diagonal due to the symmetry. An immediate consequence is that the extraordinary wave is a pure transverse wave, i.e., it is not a hybrid mode, which is in contrast to the one-component electron plasma. The comparison between the extraordinary waves in the electron-ion plasma and in the electron-positron plasma reveals that the number of cutoff frequencies and the number of resonances differ. The dispersion relation for an ordinary wave in the cold-plasma limit is found to be identical to that for a transverse wave in the absence of a magnetic field. For warm plasma, the nonlocal effects on the dispersion of the ordinary wave become important.

Finally, let us mention a few directions in which the present work may be extended. From the schemes that can produce the electron-positron plasma, it is apparent that the plasma can be either neutral or non-neutral. In the non-neutral case $(n_- \neq n_+)$, some of the features that are unique to the neutral electron-positron plasma do not hold. We thus expect that the collective mode depends on the charge excess $(n_- - n_+)$ for the electron-positron plasma much more sensitively than for the electron-ion plasma. In the present paper, we have considered the homogeneous plasma in thermal equilibrium. The present analysis may also be extended to the inhomogeneous case in order to allow for a specific plasma-confinement geometry [24,25] and/or to the nonequilibrium case, where a number of instabilities, many of which are unique to the electron-positron plasma, are expected.

In analyzing the positron annihilation lines near the direction of the Galactic Center, plasma effects have been neglected. This is justified by the low density of the positrons in most of the astrophysical environment. In the laboratory electron-positron plasma, the density may eventually reach to the degree where annihilation processes must be treated as occurring in a dielectric medium, not in a vacuum. The plasma effects can play an important role in an annihilation, where an electron with a screening cloud around it collides with a positron also with a screening cloud to annihilate. One may use the dielectric functions given in this paper to incorporate the plasma effects [26]. We wish to study and report these cases in the future.

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